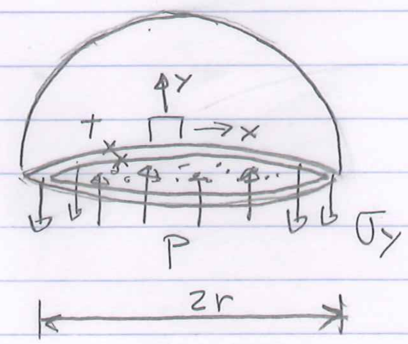
27. Application: pressure vessels

Thin-walled spherical vessel containing pressurized gas.

radius r, wall thickness t, internal pressure p, neglect weight of vessel and gas

FBD of half the vessel with gas:



Σ vertical forces $= 0$

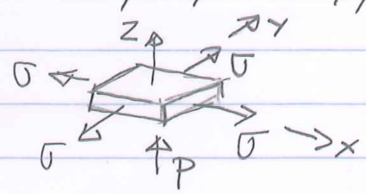
Assume σ_y is constant through the thickness.

$$\pi r^2 p - \sigma_y 2\pi r t = 0$$

$$\sigma_y = pr/2t$$

By symmetry, $\sigma_x = \sigma_y$, $\tau_{xy} = 0$

3-d stress state:



where

$\sigma_x = \sigma$, $\sigma_y = \sigma$, σ_z varies from p on inside face to zero on outside face. Since $\sigma \gg p$ due to $t \ll r$, we will neglect σ_z. Principal stresses are $\sigma_a = \sigma$, $\sigma_b = \sigma$, $\sigma_c = 0$. Thus,

$$\tau_{max} = \sigma/2 = pr/4t.$$

x-y strains in the vessel

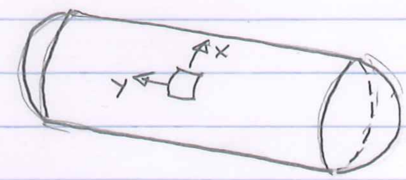
$$\varepsilon_x = \frac{\sigma_x}{E} - \frac{\nu\sigma_y}{E} - \frac{\nu\sigma_z}{E} = \frac{\sigma}{E}(1-\nu) = \frac{pr}{2t}\,\frac{1-\nu}{E}$$

$$\varepsilon_y = \ldots = \frac{pr}{2t}\,\frac{1-\nu}{E}$$

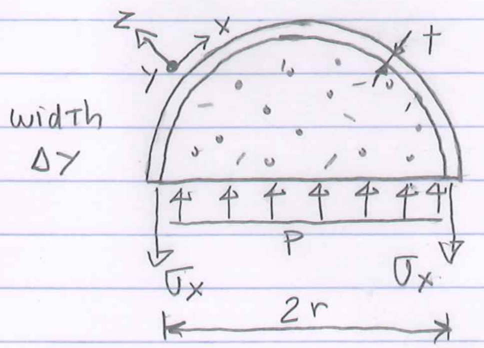
$$\gamma_{xy} = 0$$

Thin-walled cylindrical tank with spherical end caps containing pressurized gas

radius r, wall thickness t, internal pressure p, neglect weight of tank and gas



FBD of a half slice with gas:



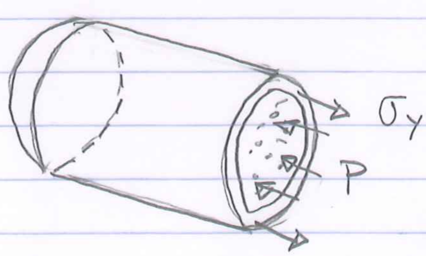
Σ vertical forces $= 0$

Assume σ_x is constant through the thickness.

$$2rp\,\Delta y - 2\sigma_x t\,\Delta y = 0$$

$$\sigma_x = pr/t$$

FBD of half tank with gas



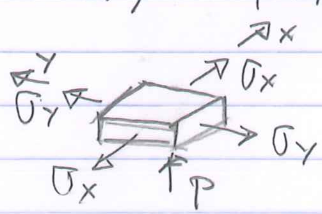
Σy forces $= 0$

Assume σ_y is constant through the thickness.

$$-\pi r^2 p + \sigma_y 2\pi r t = 0$$

$$\sigma_y = pr/2t$$

3-d stress state



where σ_x and σ_y are given above, and σ_z varies from p on inside face to zero on outside face. Since $\sigma_x, \sigma_y \gg p$ due to $t \ll r$, we will neglect σ_z.

Principal stresses are $\sigma_a = \sigma_x$, $\sigma_b = \sigma_y$ and $\sigma_c = 0$.
Thus, $\tau_{max} = \sigma_a/2 = pr/2t$.

x-y strains within the cylindrical part of the tank.

$$\varepsilon_x = \frac{\sigma_x}{E} - \frac{\nu\sigma_y}{E} - \frac{\nu\sigma_z}{E} = \frac{pr}{2t}\left(\frac{2-\nu}{E}\right)$$

$$\varepsilon_y = \frac{\sigma_y}{E} - \frac{\nu\sigma_x}{E} - \frac{\nu\sigma_z}{E} = \frac{pr}{2t}\left(\frac{1-2\nu}{E}\right)$$

$$\gamma_{xy} = 0$$

Note that the ε_x strains along the interface between the cylinder and spherical cap are incompatible. This will produce extra stresses near this interface due to bending and shearing. Such topics are covered in shell theory. (I am comparing the ε_x above with that from the spherical pressure vessel.